Solitons of Cubic-Quintic Nonlinear Schrödinger Equation with Various External Potentials

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Abstract:
Solitons are nonlinear wave structures that are widely present in nature. In nonlinear optics, localized solitary waves are usually called solitons and their existence in homogeneous and/or periodic systems is shown experimentally. In recent years, optical lattice solitons in Kerr media are deeply analyzed and it is shown that they suffer from collapse due to self-focusing or diffraction.

In this study, the existence and stability of optical solitons on periodic and certain type of quasicrystal lattices (Penrose-5 and Penrose-7) are investigated. The governing equation for the physical model that has been used is the cubic-quintic nonlinear Schrödinger equation (CQNLS) with an external potential in (2+1)D space:

$$iu_z + u_{xx} + u_{yy} + \alpha u|u|^2 + \beta u|u|^4 + Vu = 0\ . \tag{1}$$

The solution to the CQNLS equation with an external potential is obtained by spectral methods. The investigated potentials are of the following form:

$$V_N(x,y) = \frac{V_0}{N^2}\left|\sum_{n=0}^{N-1} e^{i(x\cos\frac{2\pi n}{N} + y\cos\frac{2\pi n}{N})}\right|^2 . \tag{2}$$

As the external potential V, periodic and quasicrystal lattices corresponding to $N = 4$, $N = 5$ and $N = 7$ are considered. In particular, quasicrystals for $N = 5$ and $N = 7$ are called Penrose type lattices.

For the numerical solution of Eq.(1), a Fourier iteration method, namely the spectral renormalization method is employed.

In this work, the numerical existence of fundamental solitons on the periodic and quasicrystal lattices is shown and the band-gap structures are found for varying parameters α, β and V_0.

Next, the nonlinear stability properties of the solitons are investigated and the effect of the external potential as a collapse arrest mechanism is discussed.

References:

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